To prove: $a<b\wedge a,b\in\mathbb{N}:a\nmid b\Rightarrow\forall 0\le n,m<b(n,m\in\mathbb{N}):n\neq m\Rightarrow\neg(an\equiv am(mod\,b))$ (not equivalent doesnt exist in the formula editor).
Proving by using a contraposition:
$\exists 0\le n,m<b:n\neq m\wedge an\equiv am(mod\,b)\Rightarrow a|b$
$an\equiv am(mod\,b)\Leftrightarrow an-am=a(n-m)\equiv 0(mod\,b)$
$a(n-m)|b\Rightarrow a|b\wedge(n-m)|b$
Now, I need to prove $\forall a,b,c\in\mathbb{N}:ab|c\Rightarrow a|c\wedge b|c$ and then I'm done:
$ab|c\Rightarrow\exists q\in\mathbb{Z}:q(ab)=c$
$\begin{cases}\Leftrightarrow(qa)b=c\Leftrightarrow b|c\\ \Leftrightarrow(qb)a=c\Leftrightarrow a|c\end{cases}$
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